1ª Lista de Exercícios de Cálculo II

Funções Vetoriais: Operações, Limite e Continuidade

Exercício 1 *Dadas as funções* $\vec{f}(t) = (t, 1, t^2)$, $\vec{g}(t) = \left(t-1,\ t^2,\ t^2-1\right)$ *e* $\vec{h}(t) = \left(e^t, t^2+2, 0\right)$*, calcule:*

a) $\vec{f}(t)+\vec{g}(t)$	*b)* $\vec{f}(t)+\vec{h}(t)$	*c)* $\vec{g}(t)+\vec{h}(t)$
d) $\vec{f}(t)-\vec{g}(t)$	*e)* $\vec{f}(t)-\vec{h}(t)$	*f)* $\vec{g}(t)-\vec{h}(t)$
g) $\vec{f}(t).\vec{g}(t)$	*h)* $\vec{f}(t).\vec{h}(t)$	*i)* $\vec{g}(t).\vec{h}(t)$
j) $\vec{f}(t)\times\vec{g}(t)$	*k)* $\vec{f}(t)\times\vec{h}(t)$	*l)* $\vec{g}(t)\times\vec{h}(t)$

Exercício 2 *Calcule:*

a) $\lim\limits_{t\to 2}\left(t, t^2, t^3\right)$

b) $\lim\limits_{t\to 0}(t+1, t-1, sen(t.\pi))$

c) $\lim\limits_{t\to -3}(5,\ 3,\ 2)$

d) $\lim\limits_{t\to 0}\left(\frac{t}{t^2+t}, \frac{t^2}{2t+1}, \frac{2t^2-3t+1}{3t^2+2t^2-5t+2}\right)$

e) $\lim\limits_{t\to 3}\left(\frac{t^2-9}{t^3-27}, \frac{t^2-6t+9}{t-3}, \frac{t-3}{t^2+t-9}\right)$

f) $\lim\limits_{t\to 2}\left(\frac{\sqrt{t}-\sqrt{2}}{t-2}, \frac{\sqrt{t+2}-2}{t-2}, \frac{\sqrt{t^2+2}-\sqrt{6}}{\sqrt{t}-\sqrt{2}}\right)$

Exercício 3 *Dadas as funções* $\vec{f}(t) = (t, 1, t^2)$, $\vec{g}(t) = \left(t+1, \frac{t^2-4}{t-2}, 5t-4\right)$ *e* $\vec{h}(t) = \left(e^t, t^2+2, 0\right)$*, calcule:*

a) $\lim\limits_{t\to 0}\left(\vec{f}(t)+\vec{g}(t)\right)$	*b)* $\lim\limits_{t\to 0}\left(\vec{f}(t)+\vec{h}(t)\right)$
c) $\lim\limits_{t\to 0}\left(\vec{g}(t)+\vec{h}(t)\right)$	*d)* $\lim\limits_{t\to 0}\left(\vec{f}(t)-\vec{g}(t)\right)$
e) $\lim\limits_{t\to 0}\left(\vec{f}(t)-\vec{h}(t)\right)$	*f)* $\lim\limits_{t\to 0}\left(\vec{g}(t)-\vec{h}(t)\right)$
g) $\lim\limits_{t\to 0}\left(\vec{f}(t).\vec{g}(t)\right)$	*h)* $\lim\limits_{t\to 0}\left(\vec{f}(t).\vec{h}(t)\right)$
i) $\lim\limits_{t\to 0}\left(\vec{g}(t).\vec{h}(t)\right)$	*j)* $\lim\limits_{t\to 0}\left(\vec{f}(t)\times\vec{g}(t)\right)$
k) $\lim\limits_{t\to 0}\left(\vec{f}(t)\times\vec{h}(t)\right)$	*l)* $\lim\limits_{t\to 0}\left(\vec{g}(t)\times\vec{h}(t)\right)$

Exercício 4 *Dada a seguinte definição: Uma função vetorial* $\vec{f} = \vec{f}(t)$*, definida em um intervalo I, é contínua em* t_0*, se*

$$\lim_{t\to t_0}\vec{f}(t) = \vec{f}(t_0).$$

Verifique se a função vetorial $\vec{f}$ *é contínua no ponto* $t = t_0$*, onde*

a) $\vec{f}(t) = \left(t, \frac{t+1}{t-1}, \frac{2t+3}{2t}\right)$ e $t_0 = 2$.

b) $\vec{f}(t) = \begin{cases} \left(\frac{t-3}{t^2-9}, 5\pi, \frac{t^2+2t-15}{t^2-4t+3}\right), & se\ t \neq 3 \\ \left(\frac{1}{6},\ 5\pi,\ 4\right), & se\ t = 3 \end{cases}$ e $t_0 = 1$.

c) $\vec{f}(t) = \begin{cases} \left(\frac{t-3}{t^2-9}, 5\pi, \frac{t^2+2t-15}{t^2-4t+3}\right), & se\ t \neq 3 \\ \left(\frac{1}{6},\ 5\pi,\ 4\right), & se\ t = 3 \end{cases}$ e $t_0 = 3$.

d) $\vec{f}(t) = \begin{cases} \left(\frac{t^2+2t-3}{2t-2},\ \frac{\sqrt{t}-1}{t-1}, \frac{t^3+2t-3}{t^4+t^3-2t^2}\right), & se\ t \neq 1 \\ \left(2, \frac{1}{2}, 2\right), & se\ t = 1 \end{cases}$ e $t_0 = 1$.

e) $\vec{f}(t) = \begin{cases} \left(\frac{t^2+2t-3}{2t-2},\ \frac{\sqrt{t}-1}{t-1}, \frac{t^3+2t-3}{t^4+t^3-2t^2}\right), & se\ t \neq 1 \\ \left(2, \frac{1}{2}, \frac{5}{3}\right), & se\ t = 1 \end{cases}$ e $t_0 = 1$.

Respostas

1. $a)\ \vec{f}(t)+\vec{g}(t)=(2y-1,t^2+1,2t^2-1)$ $b)\ \vec{f}(t)+\vec{h}(t)=(t+e^t,t^2+3,t^2)$
$c)\ \vec{g}(t)+\vec{h}(t)=(e+t-1,2t^2+2,t^2-1)$ $d)\ \vec{f}(t)-\vec{g}(t)=(1,1-t^2,1)$
$e)\ \vec{f}(t)-\vec{h}(t)=(t-e^t,-t^2-1,t^2)$ $f)\ \vec{g}(t)-\vec{h}(t)=(-e^t+t-1,-2,t^2-1)$
$g)\ \vec{f}(t).\vec{g}(t)=t^4+t^2-t$ $h)\ \vec{f}(t).\vec{h}(t)=t.e^t+t^2+2$
$i)\ \vec{g}(t).\vec{h}(t)=t.e^t-e^t+t^4+2t^2$ $j)\ \vec{f}(t)\times\vec{g}(t)=(t^3-t+1,t-t^2,-t^4+t^2-1)$
$k)\ \vec{f}(t)\times\vec{h}(t)=(-t^4-2t^2,t^2.e^t,t^3+2t-e^t)$ $l)\ \vec{g}(t)\times\vec{h}(t)=(t^4+t^2-2,-t^2.e^t+e^t,-t^2.e^t+t^3-t^2+2t-2)$

2. $a)\ (2,4,8)$ $b)\ (1,-1,0)$ $c)\ (5,3,2)$ $d)\left(1,0,\frac{1}{2}\right)$ $e)\ \left(\frac{2}{9},0,0\right)$ $f)\left(\frac{1}{2\sqrt{2}},\frac{1}{4},\frac{4\sqrt{2}}{\sqrt{6}}\right)$

3) $a)\ (1,3,-4)$ $b)\ (1,3,0)$ $c)\ (2,4,-4)$ $d)\ (-1,-1,4)$ $e)\ (-1,-1,0)$ $f)\ (0,0,-4)$
$g)\ 2$ $h)\ 2$ $i)\ 2$ $j)(-4,0,-1)$ $k)\ (0,0,-1)$ $l)\ (8,-4,0)$

4) Apenas a função $\vec{f}(t)$ do itêm $d)$ não é contínua no ponto indicado.